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## **Introducing A Robust Algorithm Tool for Predicting Future Performance of Oil Reservoirs**

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### **Abstract**

The objective of this work is to introduce a robust algorithmic tool for predicting the future performance of multi-sector oil reservoirs under formation-damage conditions using material balance principles. The proposed approach aims to improve the reliability of performance forecasting by estimating key reservoir parameters such as original oil in place (OOIP), aquifer strength, gas-cap size, and transmissibility between reservoir sectors while accounting for production variations caused by formation damage. The method is designed to utilize limited field data, primarily production history and fluid properties, which makes it suitable for early-stage reservoir evaluation and rapid performance assessment.

To achieve this objective, a nonlinear optimization algorithm is developed and integrated into a material balance framework. The method employs nonlinear regression techniques and an iterative Newton–Jacobian scheme to minimize the cost function between observed and calculated reservoir performance parameters. The algorithm is implemented in the latest version of the indigenous material balance software Libra, enabling efficient history matching and parameter estimation for both single-sector and multi-sector reservoir systems.

The developed algorithm enables reliable prediction of future reservoir behavior, including average reservoir pressure, oil, gas, and water production rates, aquifer water influx, and inter-sector flow. The approach has been validated using production data from several reservoirs, demonstrating high computational efficiency and stable convergence with a limited number of iterations. The results show that the proposed method can effectively handle reservoirs experiencing formation damage and significant variations in oil production rates.

This study presents a practical and computationally efficient tool for forecasting the performance of multi-sector oil reservoirs under formation-damage conditions using material balance analysis. The proposed algorithm enhances prediction stability, reduces computational effort, and provides reservoir engineers with an effective approach for reservoir management and production forecasting in complex reservoir systems.

## 1. Introduction

The input information for material balance analysis comprises pressure-decline trends with respect to time, production history data for gas, oil, and water, reservoir fluid property data, and reservoir injection data. These data are used during history matching and regression to estimate original oil in place (OOIP), gas-cap volume, aquifer properties, and the inter-sector transmissibility coefficient. In material balance analysis, it is assumed that at any time during production, the reservoir pressure is uniform throughout the reservoir. Another assumption is that the fluid properties are homogeneous across the reservoir. Therefore, any pressure differences at various reservoir locations are considered negligible.

In this method, two fundamental conditions must be satisfied to ensure the reliability of the calculated reservoir and aquifer parameters. The first condition is that the production data must be sufficiently extensive and of acceptable quality. The second condition is the ability to define a trend for the variation of average reservoir pressure. In a large reservoir, the simplest approximation is to treat the reservoir as a single tank. Under this assumption, if the reservoir pressure at all locations is referenced to a common datum, the pressure should be identical everywhere. If a consistent pressure trend can be defined, it can be used to calculate fluid properties and aquifer characteristics. In addition, when applying fluid property data, it is assumed that laboratory measurements accurately represent actual reservoir conditions. One important consideration in material balance calculations is that differential liberation test data are assumed to represent the reservoir flow conditions, while flash (separator) test data are used to correct downhole conditions to surface conditions.

Multi-sector oil reservoirs refer to reservoirs in which multiple pressure-decline trends can be identified. These sectors, although exhibiting different pressure trends, remain hydraulically connected, allowing fluid flow between them. In practice, calculating inter-sector flow can be complex. For example, in a two-sector reservoir with overlapping drainage radii and asymmetric well placement, flow may simultaneously occur between both sectors in opposite directions. However, the dominant flow generally occurs from the sector with higher pressure to the sector with lower pressure. The material balance method calculates the dominant flow at each time step as a function of the inter-sector pressure difference, the sum of relative permeability-to-viscosity ratios of the three phases in the upstream sector, and the inter-sector transmissibility coefficient.

For predicting future reservoir performance using material balance, the actual oil production rate from each sector is calculated through inflow performance relationships (IPR). The cost function for each sector is defined as the difference between the actual oil production rate and the oil production rate calculated through material balance for that sector. By minimizing this cost function—which is a nonlinear function dependent on sector pressure and phase saturations—the average reservoir pressure, gas and water production rates, aquifer water influx, and inter-sector flow rates can be estimated. In this paper, the algorithm for predicting future performance of single-sector and multi-sector reservoirs, implemented in the latest version of the indigenous material balance software *Libra*, is described. Verification results are presented for forecasting the future performance of an oil reservoir in southwest Iran and a synthetic two-sector reservoir model.

## 2. Material Balance of Multi-Sector Reservoirs

The material balance equation was first introduced by Schilthuis in 1941, in which the reservoir was considered equivalent to a single tank (sector). In this equation, the initial volume of fluids in each sector is equal to the sum of the remaining fluid volume and the volume of fluids produced from that sector. The general material balance equation for each sector of a multi-sector reservoir is expressed as follows:

$$N = \frac{N_p(B_o + (R_p - R_s)B_g) - (W_e - W_p B_w) - G_{inj}B_{g,inj} - W_{inj}B_{w,inj} \pm flow}{B_o - B_{oi} + (R_{si} - R_s)B_g + mB_{oi} \left( \frac{B_g}{B_{gi}} - 1 \right) + (m + 1)B_{oi} \left( \frac{C_f + C_w S_{wi}}{1 - S_{wi}} \right) \Delta P} \quad (1)$$

where:

$N$  – initial oil in place;  $N_p$  – cumulative oil produced from each sector;

$\Delta P$  – pressure change in each sector;

$S_{wi}$  – initial formation water saturation;

$C_f$  – reservoir rock compressibility;  $C_w$  – reservoir water compressibility;

$B_g$  – gas formation volume factor at current pressure in each sector;

$B_{gi}$  – initial gas formation volume factor in each sector;

$m$  – ratio of gas-cap volume to oil volume at the initial pressure of each sector;

$R_s$  – solution gas–oil ratio at current pressure;  $R_{si}$  – solution gas–oil ratio at initial pressure;

$B_o$  – oil formation volume factor at current pressure;  $B_{oi}$  – initial oil formation volume factor;

$B_w$  – water formation volume factor at current pressure;

$R_p$  – produced gas–oil ratio;

$W_e$  – cumulative water influx into the reservoir;  $W_p$  – cumulative water production;

$G_{inj}$  – cumulative gas injection;  $B_{g,inj}$  – formation volume factor of injected gas at current pressure;

$W_{inj}$  – cumulative water injection

$B_{w,inj}$  – formation volume factor of injected water at current pressure;

$flow$  – inter-sector flow rate.

To calculate the inter-sector flow between two sectors, the following relationship is used:

$$flow^{n+1} = flow^n + C * \Delta P * \Delta t * \sum_i^{n_p} \frac{k_{ri}}{\mu_i} \quad (2)$$

where:

$flow^{n+1}$  – inter-sector flow rate at the new time step;  $flow^n$  – inter-sector flow rate at the previous time step;

$C$  – transmissibility coefficient;

$\Delta P$  – pressure difference between the two sectors;  $\Delta t$  – time-step size;

$\sum_i^{n_p} \frac{k_{ri}}{\mu_i}$  – sum of the relative permeability–to–viscosity ratios of the three phases (water, oil & gas).

## 2.1 Steps of Material Balance Study

In a proper material balance study, before performing history matching, a sensitivity analysis is conducted on the regression parameters to determine their optimal upper and lower bounds. After completing the history matching process, the accuracy of the obtained results is evaluated. For this purpose, using known values of original oil in place, gas-cap volume, aquifer properties, and inter-sector transmissibility, the pressure of each sector is predicted and compared with the observed pressure data.

After validating the history matching results, a prediction analysis is performed from the beginning of the historical data to the end of the historical period. During this step, the relative permeability curves are adjusted so that the predicted water and gas production match the observed water and gas production data. Once appropriate relative permeability curves are obtained, a future prediction analysis is carried out from the end of the historical data to any desired future time.

In the prediction stage, by minimizing the cost function for each sector—defined as the absolute difference between the actual oil production from the sector and the oil production calculated using the material balance method—the average reservoir pressure, gas and water production rates, aquifer water influx, and inter-sector flow rates are determined. The cost function, which is a nonlinear function dependent on the pressure and phase saturations of each sector, is presented in the following section.

## 2.2 Cost Function in Predicting the Performance of Single-Sector and Multi-Sector Reservoirs

The absolute difference between the oil production calculated from material balance,  $N_p^{Cal}$ , and the actual oil production from each sector,  $N_p^{Real}$ , which is obtained from the IPR relationship, is defined as the cost function. By defining the denominator of Equation (1) as DEN, the cost function for each sector is obtained according to Equation (4).

$$DEN = B_o - B_{oi} + (R_{si} - R_s)B_g + mB_{oi} \left( \frac{B_g}{B_{gi}} - 1 \right) + (m + 1)B_{oi} \left( \frac{C_f + C_w S_{wi}}{1 - S_{wi}} \right) \Delta P \quad (3)$$

$$COST \text{ Function} = |N_p^{Cal} - N_p^{Real}| \\ = \left| \frac{N \cdot DEN + (W_e - W_p B_w) + G_{inj} B_{g,inj} + W_{inj} B_{w,inj} \pm flow}{(B_o + (R_p - R_s)B_g)} - N_p^{Real} \right| \quad (4)$$

The cost function for each sector depends on the average sector pressure, time, water production, gas–oil ratio, and inter-sector flow. However, since water production, gas–oil ratio, and inter-sector flow are themselves dependent on the pressure and phase saturations of water, oil, and gas in each sector, the cost function, according to Equation (5), ultimately depends on the average pressure of each sector, time, and the phase saturations present in each sector.

$$COST \text{ Function} = f(P, t, R_p, W_p, flow) = f(P, t, S_o, S_w) \quad (5)$$

To calculate the actual oil production from each sector and the amount of water and gas injected into each sector, inflow performance relationship (IPR) equations are used. For calculating the oil production rate in producing wells when the constraint defined for prediction is the production rate, the bottomhole pressure is first calculated using the IPR equation according to Equation (7). If the calculated bottomhole pressure is greater than the minimum bottomhole pressure specified for the prediction, then the actual production rate is equal to the rate specified for the prediction.

If the calculated bottomhole pressure is lower than the specified minimum bottomhole pressure, the prediction constraint is automatically changed to a pressure constraint, and the production rate is calculated using the minimum bottomhole pressure through Equation (6). Similarly, if the constraint initially defined for the prediction is pressure, the production rate is calculated using the specified bottomhole pressure through Equation (6).

$$q_o = PI(\bar{P} - P_{Bh}) \frac{\frac{Kro}{\mu_o}}{\sum \frac{Kr}{\mu}} \quad (6)$$

$$P_{Bh} = \bar{P} - \frac{q_o}{PI * \frac{\frac{Kro}{\mu_o}}{\sum \frac{Kr}{\mu}}} \quad (7)$$

where:

$P_{Bh}$  – bottomhole pressure;  $\bar{P}$  – average sector pressure;

$\frac{Kro}{\mu_o}$  – ratio of oil relative permeability to oil viscosity; and

$\sum \frac{Kr}{\mu}$  – sum of the relative permeability-to-viscosity ratios of the oil, water, and gas phases.

$$N_p^{Real} = q_{o,total} \times \Delta t \quad (8)$$

To calculate the water injection rate ( $W_{inj}$ ) and gas injection rate ( $G_{inj}$ ) when the constraint defined for prediction is the production/injection rate, the bottomhole pressure is first calculated using the water and gas IPR equations. If this calculated bottomhole pressure is less than the maximum bottomhole pressure specified for the prediction, then the injection rate is equal to the rate defined for the prediction.

If the calculated bottomhole pressure exceeds the specified maximum bottomhole pressure, the prediction constraint is automatically changed to a pressure constraint, and the injection rate is calculated using the maximum bottomhole pressure through Equations (9) and (10). Similarly, if the constraint for prediction is initially defined as pressure, the injection rate is calculated using the specified bottomhole pressure through Equations (9) and (10).

$$q_{w,inj} = PI(P_{BH} - \bar{P}) \quad (9)$$

$$q_{g,inj} = c(P_{BH}^2 - \bar{P}^2)^n \quad (10)$$

In Equations (9) and (10):

$q_{w,inj}$  – water injection rate

$q_{g,inj}$  – gas injection rate

$c$  &  $n$  – constant parameters of the IPR equation

After calculating the water and gas injection rates for each well individually, the total water injection rate ( $q_{w,inj,total}$ ) and total gas injection rate ( $q_{g,inj,total}$ ) for each sector are obtained. The sector-level water and gas injections are then calculated according to Equations (11) and (12).

$$W_{inj} = q_{w,inj,total} \times \Delta t \quad (11)$$

$$G_{inj} = q_{g,inj,total} \times \Delta t \quad (12)$$

### 3. Reservoir Performance Prediction

Unlike conventional reservoir simulators, which divide the reservoir into blocks and solve flow equations, the material balance method treats the reservoir as a single tank and assigns a uniform pressure to the entire reservoir. Naturally, this assumption introduces some error during prediction. However, if the material balance conditions are satisfied—meaning the pressure-decline trend is uniform across all points within a sector—material balance can predict reservoir performance with reasonable accuracy, comparable to simulator results.

According to Equation (4), the cost function for each sector depends on the water production and gas–oil ratio of that sector. In the algorithm presented in this study, Equations (13) and (14) are used to calculate water production and the gas–oil ratio for each sector.

$$R_p = R_s + \frac{k_{rg}\mu_o B_o}{k_{ro}\mu_g B_g} \quad (13)$$

$$W_p^{n+1} = W_p^n + (N_p^{Real} - N_p^n) \frac{k_{rw}\mu_o B_o}{k_{ro}\mu_w B_w} \quad (14)$$

where:

$k_{rg}$  – relative permeability of the gas phase;

$k_{ro}$  – relative permeability of the oil phase;

$k_{rw}$  – relative permeability of the water phase;

$\mu_o$  – oil viscosity;

$\mu_g$  – gas viscosity;

$\mu_w$  – water viscosity;

$W_p^{n+1}$  – water production at the new time step;

$W_p^n$  – water production at the previous time step; and

$N_p^n$  – oil production at the previous time step.

$$S_o = \frac{(N - N_p^{Real})B_o \pm flow * F_o}{\frac{NB_{oi}}{1 - S_{wi}}(1 + m)(1 - c_f \Delta p)} \quad (15)$$

$$S_w = S_{wi} + \frac{W_e - W_p B_w + W_{inj} B_{w,inj} \pm flow * F_w}{\frac{NB_{oi}}{1 - S_{wi}}(1 + m)(1 - c_f \Delta p)} + \frac{c_w S_{wi} \Delta p}{1 - c_f \Delta p} \quad (16)$$

In addition, the gas-phase saturation is calculated using the relation  $S_g = 1 - S_w - S_o$ . In Equations (15) and (16), the terms  $F_o$  and  $F_w$  are obtained from the following relationships:

$$F_o = \frac{\frac{k_{ro}}{\mu_o}}{\sum_i^{n_p} \frac{k_{ri}}{\mu_i}} \quad (17)$$

$$F_w = \frac{\frac{k_{rw}}{\mu_w}}{\sum_i^{n_p} \frac{k_{ri}}{\mu_i}} \quad (18)$$

At the end of each time step, and after the cost function has converged, Equation (19) is used to calculate gas production. The following section describes the algorithm for predicting the performance of single-sector and multi-sector reservoirs.

$$G_p^{n+1} = G_p^n + (N_p^{Real} - N_p^n)R_p \quad (19)$$

### 3.1 Algorithm for Reservoir Performance Prediction

Fig. 1 illustrates the reservoir performance prediction algorithm for single-sector and multi-sector reservoirs, which consists of two loops: an outer (main) loop and an inner (sub) loop. In the main loop, the pressure of each sector is calculated, while in the sub-loop, the phase saturations of each sector and the inter-sector flow are determined. Fig. 2 shows the sub-loop algorithm, which is called after the pressure of each sector has been computed and operates independently of the pressure to calculate the phase saturations and inter-sector flow for each sector.

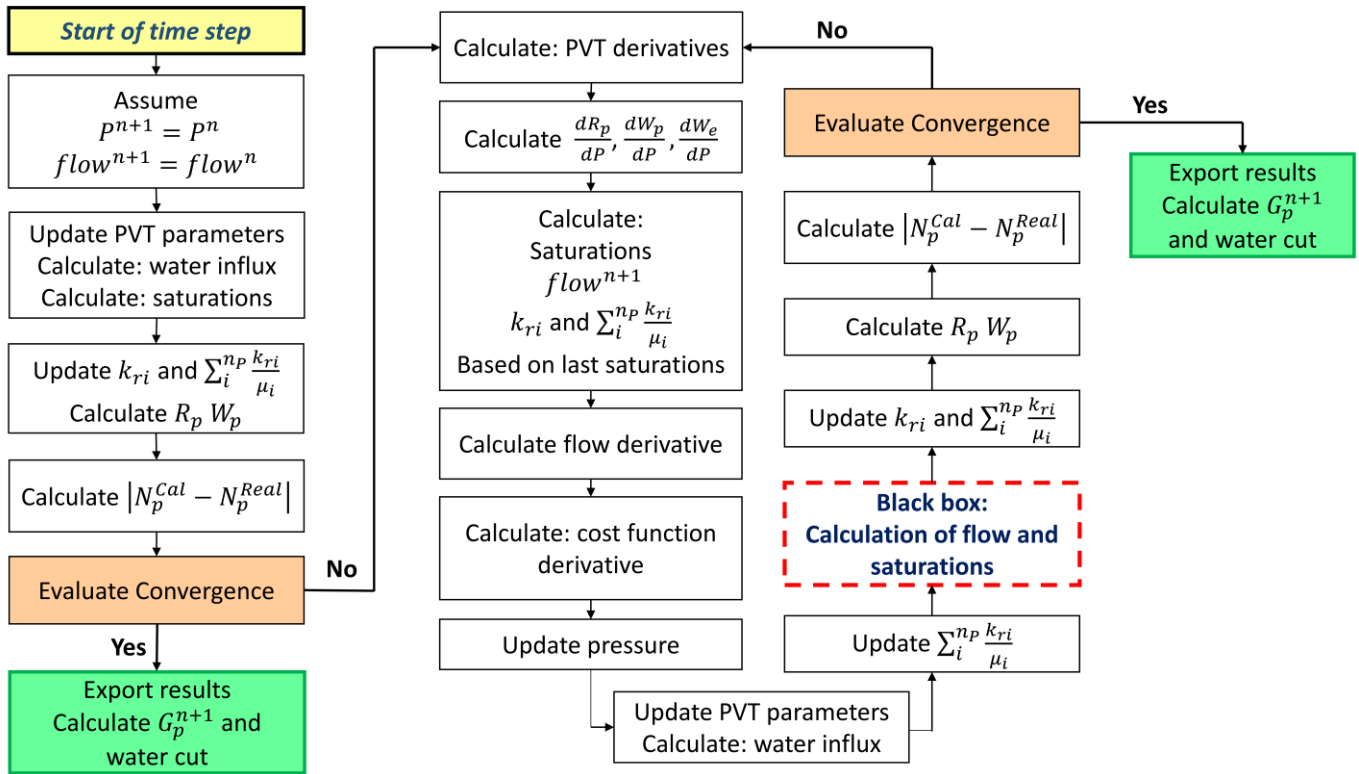


Fig. 1— Algorithm for Performance Prediction of single-sector and multi-sector reservoirs

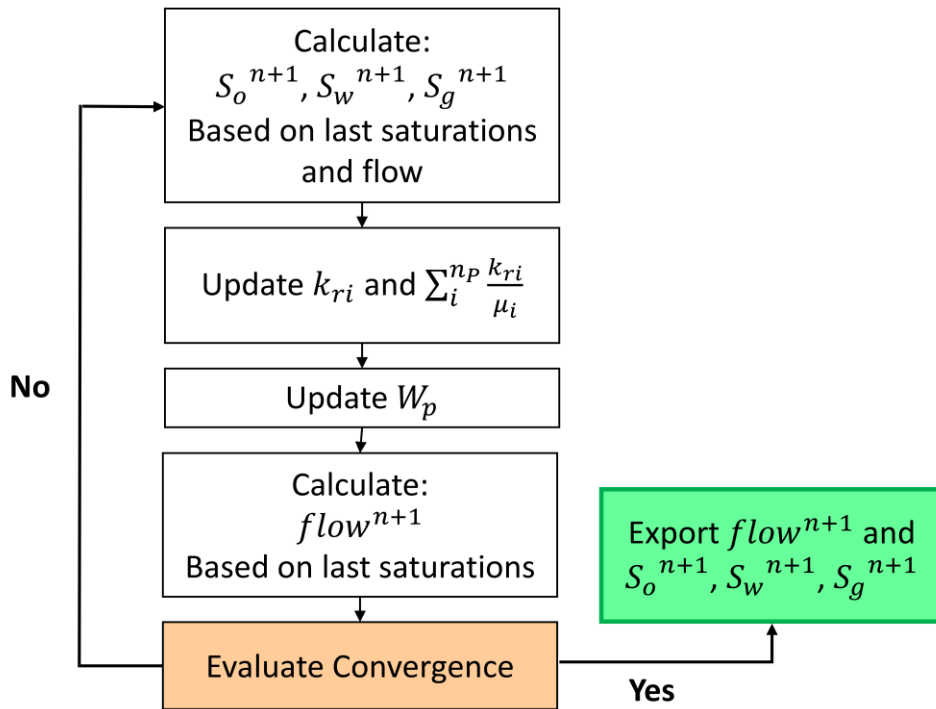


Fig. 2— Outer Loop Used in the Reservoir Performance Prediction Algorithm

### 3.2 Newton–Jacobian Iterative Method for Minimizing the Cost Function

The minimization of the cost function is performed at each time step, and the reservoir performance prediction algorithm is executed for every time step. At the beginning of each time-step, it is assumed that the pressure and the inter-sector flow rates are equal to their values in the previous time step. Then, the reservoir fluid properties, water influx, and phase saturations in each sector are calculated.

After calculating the phase saturations in each sector, the relative permeabilities of the phases, water production rate, and the gas–oil ratio (GOR) for each sector are determined. Next, the convergence of the cost function is examined. If convergence is achieved, the gas production rate is obtained from Equation (19), other required prediction outputs are calculated, and the main loop of the algorithm is terminated. However, if convergence of the cost function is not achieved, the following steps are repeated until convergence is obtained:

1. The derivatives of fluid properties, water influx, water production, and gas–oil ratio with respect to pressure are calculated separately. Equation (20) is used to compute the derivative of the water influx.
2. Based on the latest calculated saturations, the relative permeabilities, inter-sector flow rates, and phase saturations in each sector are calculated.
3. The inter-sector flow rate equation is differentiated with respect to pressure.
4. By calculating the derivative of each term of the cost function with respect to pressure, the derivative of the cost function with respect to pressure is obtained. The new pressure for each sector is then calculated using the Newton equation (21).
5. Using the newly calculated pressure, the water influx and fluid properties for each sector are recalculated.
6. Through the internal (secondary) loop—whose algorithm is presented in Figure 2—the inter-sector flow rates and the phase saturations in each sector are calculated. In this internal iterative loop, the phase saturations in each sector are first calculated based on the latest values of saturation and inter-sector flow rates. Then, using the calculated saturations, the relative permeabilities of the phases and the water production rate are determined. Subsequently, the inter-sector flow rates are calculated. The calculations in this loop are repeated until convergence in the inter-sector flow rates is achieved.
7. After determining the phase saturations in each sector, the relative permeabilities of the phases, the gas–oil ratio, and the water production are calculated.
8. The convergence of the cost function is then examined. If convergence is achieved, the gas production rate is obtained from Equation (19), and other prediction outputs—including pressure, water production, gas production, oil production, water influx, and inter-sector flow rates—are reported. The main loop of the algorithm is then terminated. However, if convergence is not achieved, Steps 1 through 8 are repeated until convergence is obtained.

$$\frac{\partial W_e}{\partial P} = \frac{W_e(P + 0.5h) - W_e(P - 0.5h)}{h} \quad (20)$$

$$P_{New} = P_{Old} - \frac{COST\ Function}{\left(\frac{\partial(COST\ Function)}{\partial P}\right)} \quad (21)$$

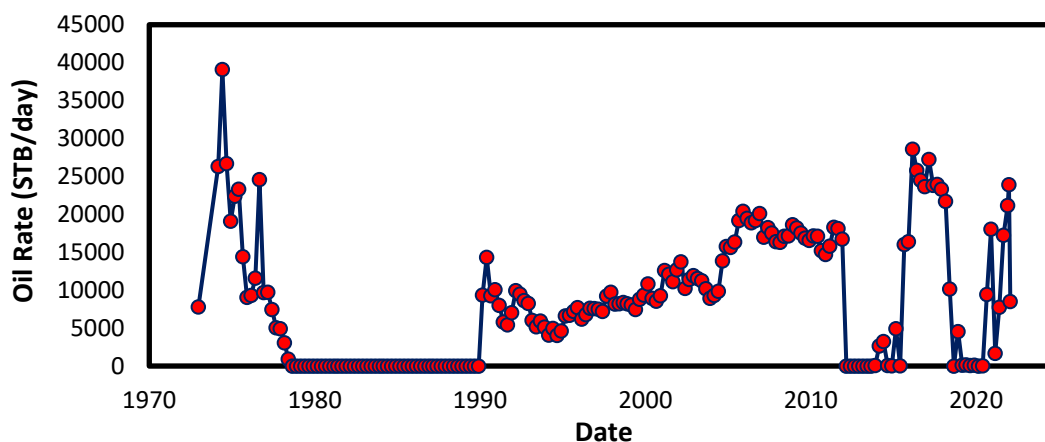
#### 4. Results and Conclusions

The first model is a real reservoir model from the Middle East, Table 1 presents its history-matching data and properties. This single-sector reservoir does not contain free gas, and its pressure increases during some time intervals due to the presence of a strong aquifer and the shut-in of wells.

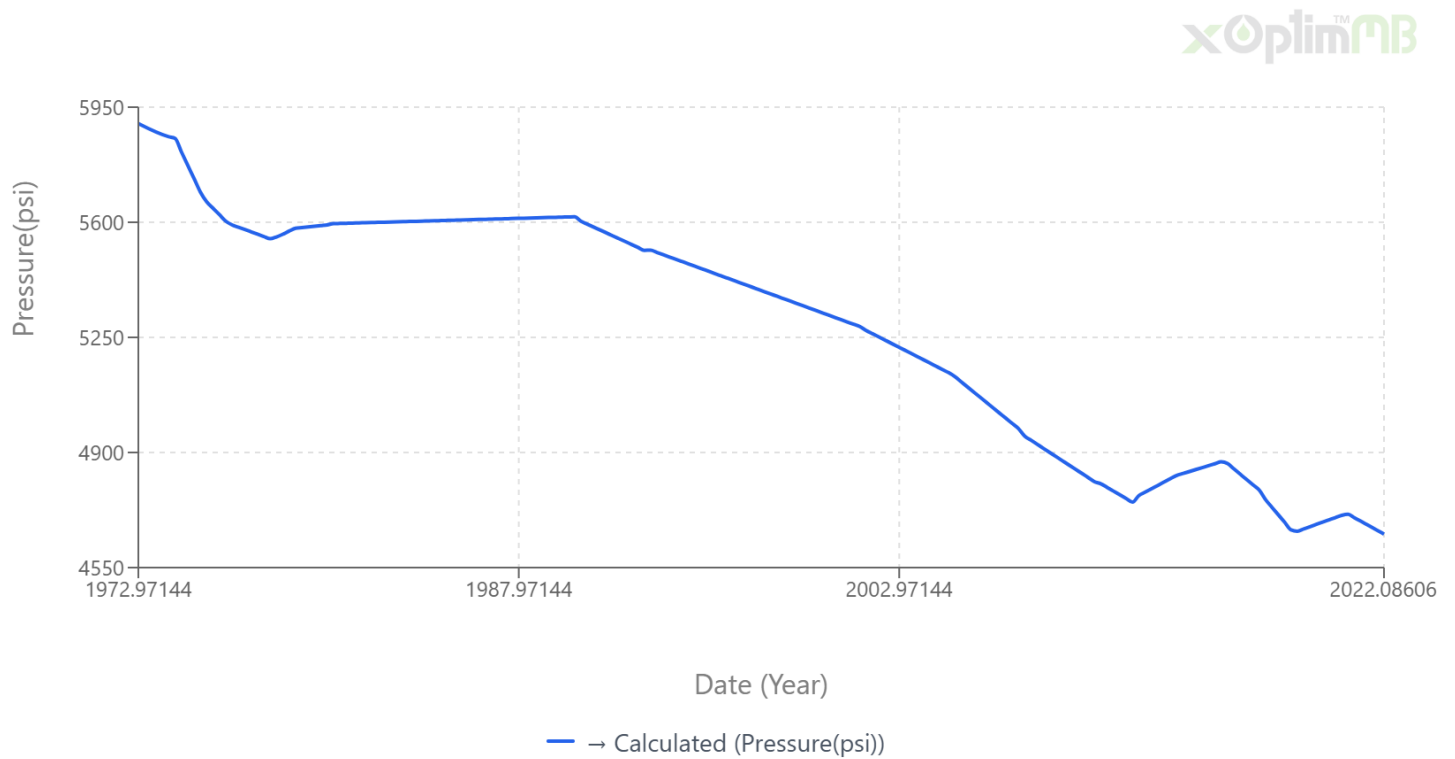
Fig. 3 shows the total production rate versus time for this model. Figures 4 through 7, respectively, show comparison results between the observed values and the predicted values of sector static pressure, oil production, water production, and gas production.

**Table 1—Basic input data for the single-sector reservoir modeling**

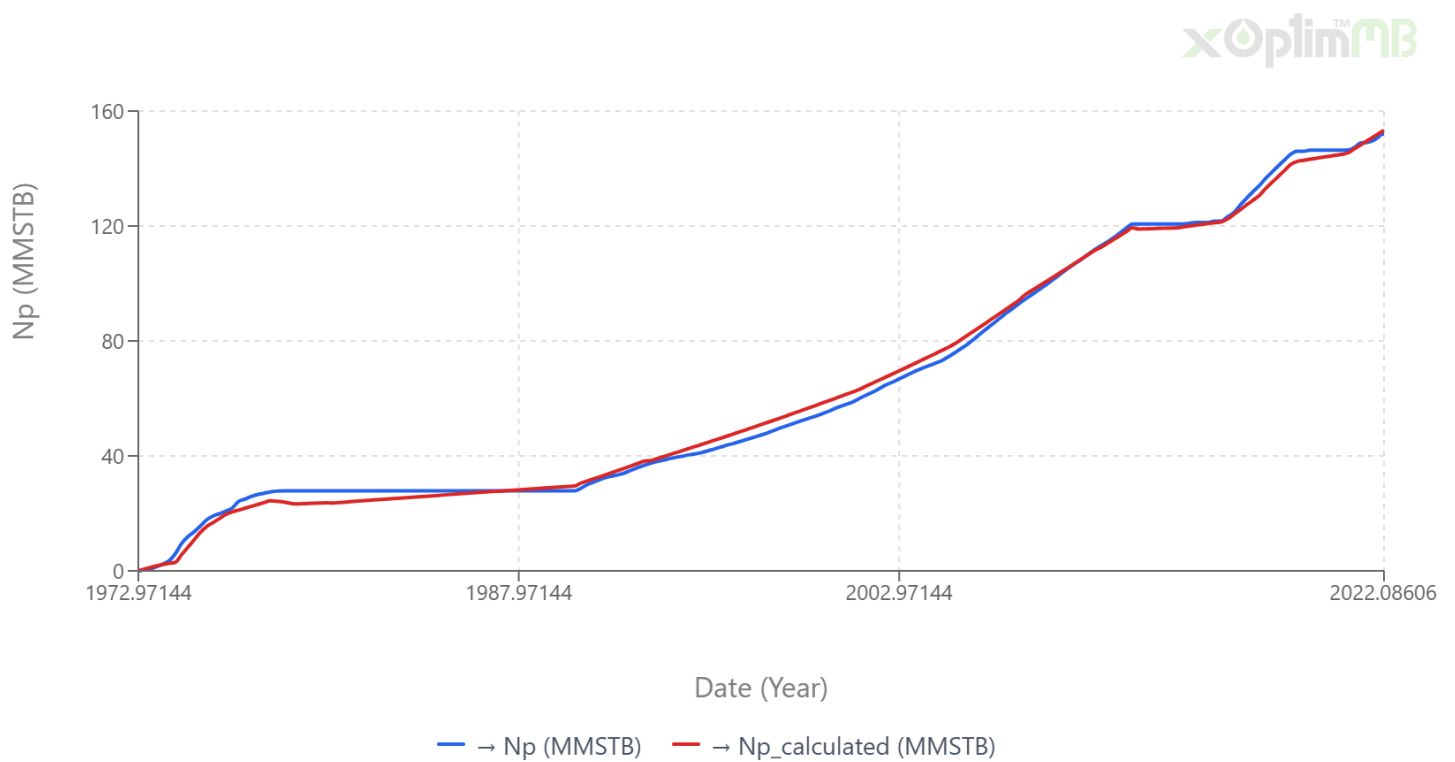
Parameter	Value	Parameter	Value
Original Oil in Place	4,619.53 MMSTB	Rock Compressibility	2e-6 1/psi
Initial Pressure	5,900 psi	Water Compressibility	6.076e-6 1/psi
Initial Solution Gas Oil Ratio	565 SCF/STB	Aquifer Porosity	0.076
Gas Cap Size/Volume Over Oil Volume (at Initial Pressure)	Zero	Total Aquifer Compressibility	4e-6 1/psi
Initial Water Saturation	0.22	Aquifer Viscosity	1 cp
Aquifer Permeability	5.3 mD	Aquifer Radius	7,225.29 ft
Aquifer Thickness	392.29 ft	Dimensionless Radius (R <sub>D</sub> )	14.76



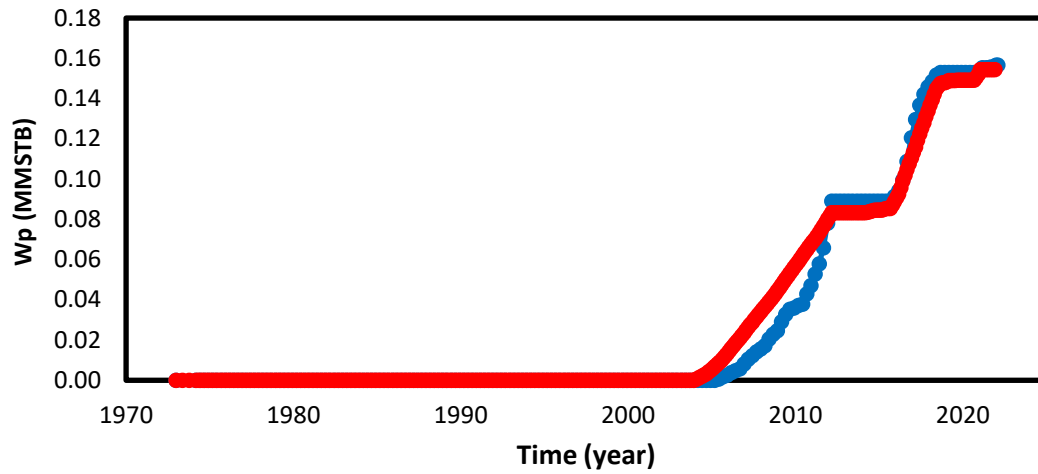
**Fig. 3 — Total well production rate of Model 1 versus time**



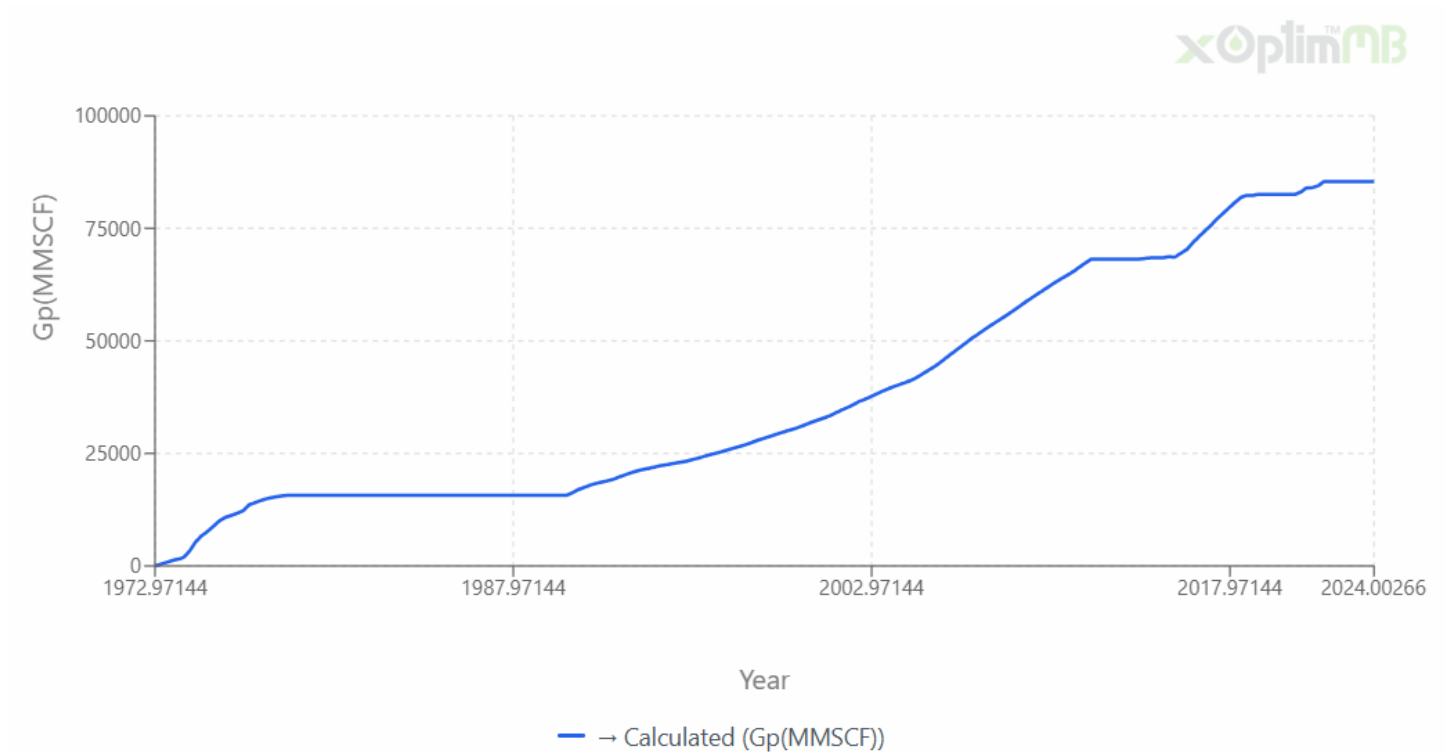
**Fig. 4 — the historical static pressure of the single-sector predicted by the innovative material balance software.**



**Fig. 5— Comparison of oil production from the single-sector model with the oil production predicted by the innovative material balance software. Note the average absolute error of only 5.78%.**



**Fig. 6— Comparison of water production from the single-sector (red color) with the water production predicted by the innovative material balance software (blue color).**



**Fig. 7— Comparison of gas production from the single-sector model with the gas production predicted by the innovative material balance software.**

## Conclusion

An innovative material balance algorithm has been developed and applied to predict future reservoir performance under conditions both with and without formation damage. To validate the algorithm, it was applied to two models—a single-sector model and a two-sector model. The results presented in this section demonstrate the high accuracy and reliability of the proposed innovative algorithm.

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